Nonlinear PDE: Burgers' Equation (Fluid Flow Model) with Dirichlet Boundary Conditions (u specified on the boundary)

$$u_t + uu_x = Du_{xx}$$
$$u_t + uu_x - Du_{xx} = 0$$

$$\frac{\overbrace{w_{ij} - \overbrace{w_{i,j-1}}^{\text{Known}}}^{\text{Backward Difference Method}}}{\underbrace{k}_{\text{Time variable}}} + \frac{\overbrace{w_{ij}\left(w_{i+1,j} - w_{i-1,j}\right)}^{\text{Center Difference Method}}}{2\ \underbrace{h}_{\text{Space Variable}}} - D\frac{w_{i+1,j} - 2w_{ij} + w_{i-1,j}}{h^2} = 0$$

$$w_{ij} - w_{i,j-1} + \frac{k}{2h}\left(w_{ij}\right)\left(w_{i+1,j} - w_{i-1,j}\right) - \frac{Dk}{h^2}\left(w_{i+1,j} - 2w_{ij} + w_{i-1,j}\right) = 0$$

$$\sigma = \frac{Dk}{h^2}$$

Simultaneous Eqns (m equations; m unknowns):

$$F_i(\vec{z}) = z_i - w_{i,j-1} + \frac{k}{2h} z_i\left(z_{i+1} - z_{i-1}\right) - \sigma\left(z_{i+1} - 2z_i + z_{i-1}\right) = 0$$
$$= z_i + \frac{k}{2h} z_i\left(z_{i+1} - z_{i-1}\right) - \sigma\left(z_{i+1} - 2z_i + z_{i-1}\right) - w_{i,j-1} = 0$$

Multivariate Newton's Method (Nonlinear Systems of Eqns):

$$x_{k+1} = x_k - \left(DF\left(x_k\right)\right)^{-1} F(x_k); \qquad k = 0,1,2,\ldots$$

Matrix Inverses

Computationally Intensive

Refashion:

$$x_{k+1} = x_k - s$$
$$DF(x_k)s = -F(x_k)$$
$$x_{k+1} = x_k + s$$